Bifurcation Analysis for a Mathematical Model to Understand Guillain-Barré Syndrome

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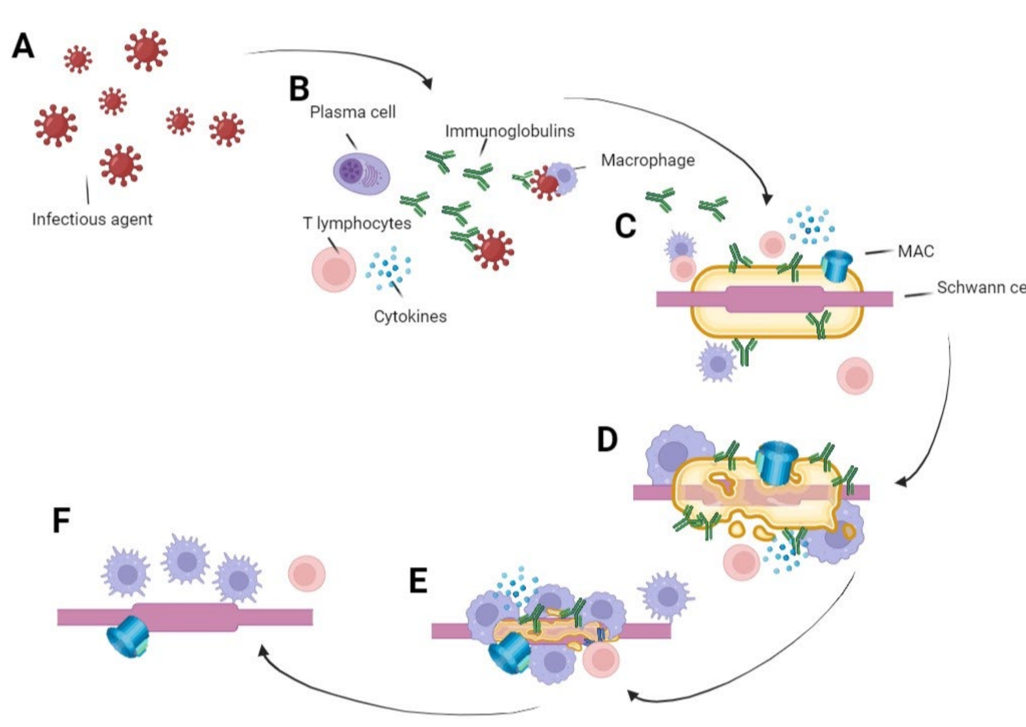
Introduction

Guillain-Barré syndrome (GBS) is an acute immune-mediated and potentially fatal polyneuropathy that develops after an infectious or immune-mediated process [1].

It leads to an exaggerated inflammatory and immune response that progressively and rapidly affects the myelin or the axon of the peripheral nerves in the human body [1-3].

GBS is the principal cause of flaccid paralysis in the world [2]. Globally, the prevalence of GBS continues to increase [4] and their incidence of this condition varies depending on the region, sex, exposure to different infections, post-vaccine effects, and genetic susceptibilities that occur worldwide. [3,5,6]



(A) Arrival of an infectious or viral agent
(B) Initiation of immune response
(C) Arrival of inflammatory and immune mediators to myelin sheaths
(D) Invasion of macrophages
(E) Vesicular degeneration
(F) Complete demyelination

Figure 1. *Pathogenesis of GBS*

Methods

The proposed basic mathematical model is defined by a system of autonomous ordinary differential equations. With three state variables corresponding to the population size of viral particles (V), immune cells (I), and Schwann cells (S), dependent on the temporal variable t, which is considered in days.

All parameters and constants r, w, β, p, μ, g, k, α are > 0.

$$\frac{dV}{dt} = rV\left(1 - \frac{V}{w}\right) - \beta VI$$

$$\frac{dI}{dt} = p\beta VI - \mu I$$

$$\frac{dS}{dt} = gS\left(1 - \frac{S}{k}\right) - \alpha SI$$

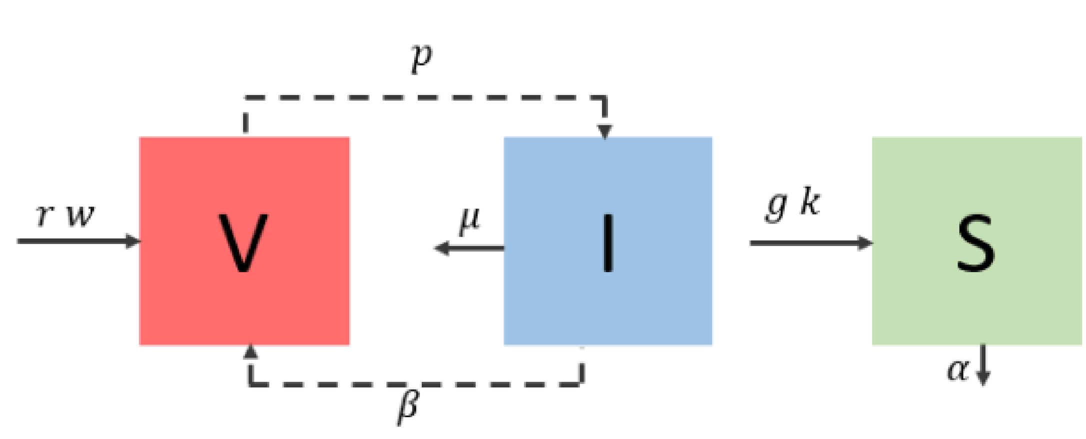


Figure 2*. The diagram blocks of the basic model*

Results

Equilibrium and stability:

Decoupling the basic model when
I = 0 $E_i = (V_i, I_i)$
Stability of the all the system when
I > 0 $E_i = (V_i, I_i, S_i)$

Stability analysis of each established by studying the eigenvalues [λ_i] of the Jacobian matrix J_i of each equilibrium. As a stability criterion was used a Routh-Hurwitz [7]. The system has 6 equilibriums: 5 stables and 1 unstable.

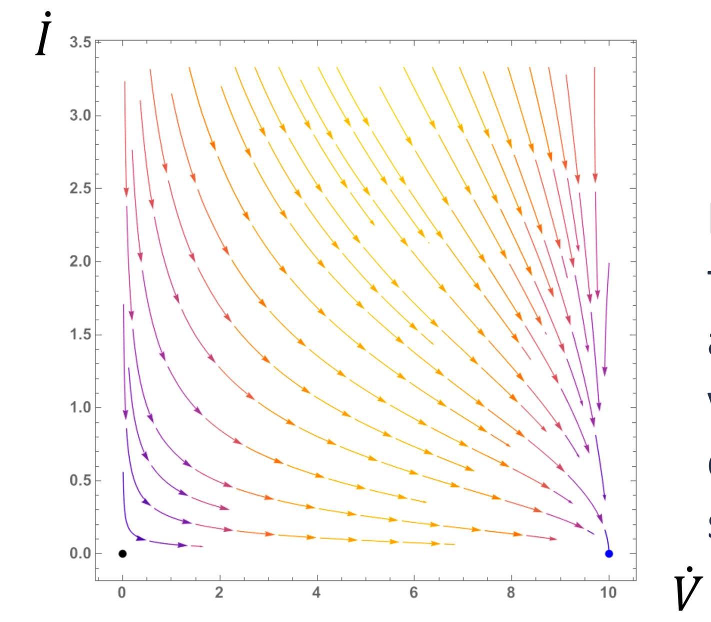


Figure 3 shows boundary equilibria, the first (black dot) is the trivial equilibrium and the second (blue dot) is when the virus grows independently to it carrying capacity, in both equilibria the immune system is not activated.

Figure 3. *Phase plane with β= 0.001*

As β increases the condition of existence of the immune system, so the system shows a new equilibrium (blue dot) which is the coexistent of the virus and de immune system and has convergence in real eigenvalues, see figure 4.

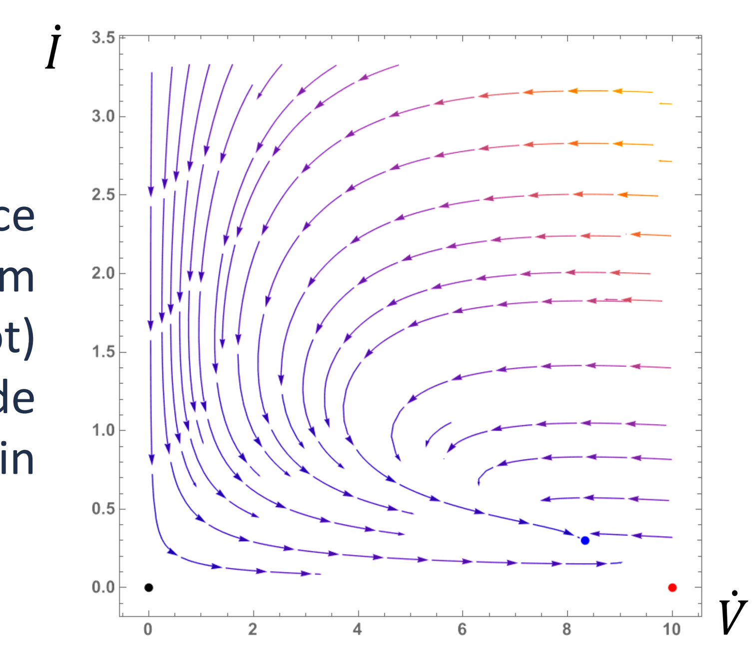
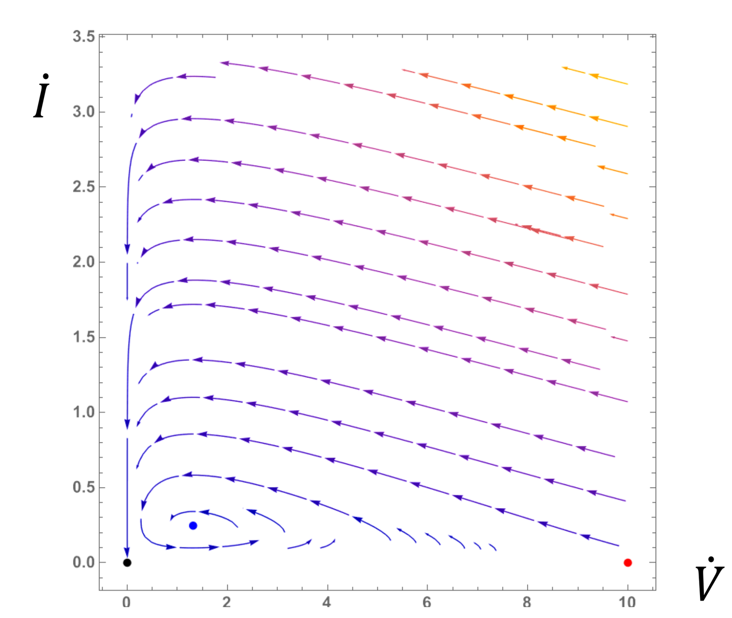


Figure 4. *Phase plane with β= 0.06*



The other dynamics happen in figure 5 when the equilibria (blue dot) have eigenvalues complex and going to converge in oscillations.

Figure 5. *Phase plane with β= 0.38*

Existence

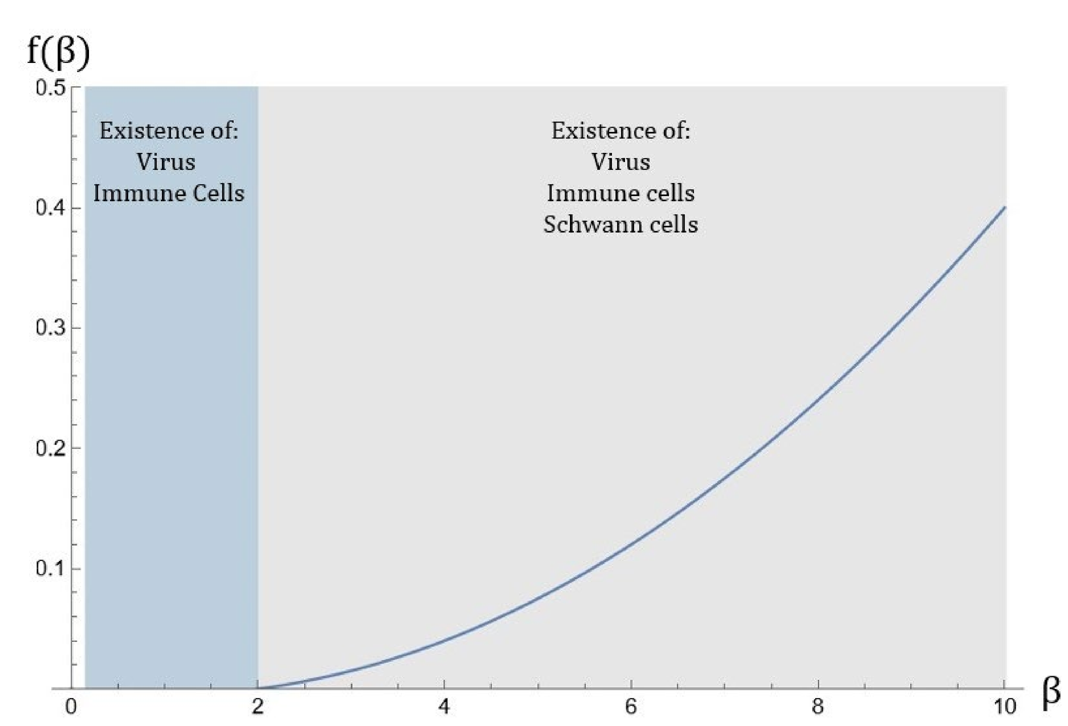


Figure 6. *Existence of the variables in the basic model*

Future Work

Analyzing the immunoglobulin model
Validate and calibrate the model

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