Analyzing the Spectral Properties of Graphs

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Background

- Helping to model the spread of invasive species, in particular the pest *Tuta absoluta* which is a tomato moth that is indigenous to South America, but, has begun spreading in both Africa as well as South East Asia.
- The spread of invasive species can be modeled using a graph where the nodes represent location and edges represent methods of spread.
- Break up the different methods of spread into 3 different components. A short distance local component, a human-mediated intra-locality component, and a long distance human-mediated spread.

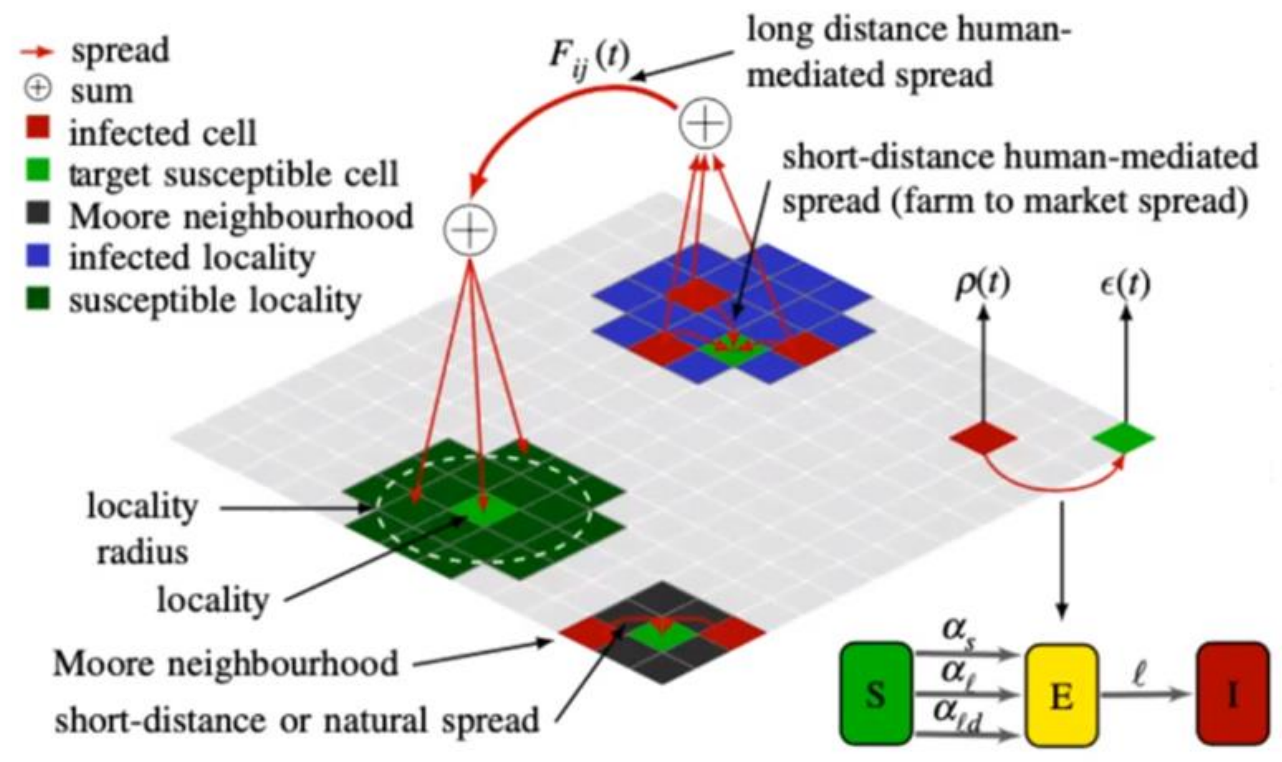

Figure #1

Results

- ANOVA tests were used to analyze the relationship between the graphs parameters and the spectral radius and the results can be found in the figure below
- Found out that there is a very strong correlation between the size of the localities and spectral radius.
- The outlier in this regard is that the correlation between range and spectral radius is much smaller than expected
- One possible explanation for this is that since the long distance connections were randomly generated there were not enough trial in getting the expected spectral radius.

ANOVA Test Results

	Sum of Squares	F statistic	P value
Locality Graph	22423	94.65	3.18E-17
Locality Size	461250	1947.5	2.74E-81
Range	1087	4.59	3.40E-2
Number of Regions	14759	62.3	9.65E-13
ER Probability	62978	265.85	1.61E-33

Figure #3

Future Work

- Generate more graphs with different parameters. The previously created graphs do not vary number of nodes so currently we do not have the data on correlation between number of nodes and the properties of the graphs.
- Repeat the analysis for different properties of the graph such as the diameter of the graph.
- Add different methods of generating both localities and long distance edges. Including both an option to generate the long distance edges from a scale-free graph and from a Chung-Lu graph could provide interesting results as it could help to better mimic real world spread. Another idea is to connect nodes within a locality using an Erdős–Rényi graph.

Current Work

- Focused on creating a theoretical, idealized graph that could be used to represent the spread of any invasive species. The graph was broken up into equally sized localities spaced in a grid around the graph connected to one another via long distance edges.
- The graph was parameterized in order to measure how different graph structure affects different properties of the graphs
- Properties of this graph are directly related to the spread of the contagion or in this case the invasive species
- In particular the spectral radius of the adjacency matrix of this graph indicates how fast the infection will spread, and whether the or not the infection will die out or infect every single node.

Design

- Generated graphs with different number of nodes, regions, locality size and type, and long distance generation
- The long distance connections were generated randomly using an Erdős–Rényi graph to model the connections between each of the localities.
- The probability that there was an edge between any two localities was also one of the parameters that was varied to find out its impact on the spectral radius.

Experiment

- Once the graphs were parameterized tests were ran by creating different graphs and altering their parameters slightly and recording the properties of the graphs.
- After generation the data was then analyzed to find correlation between the parameters and the properties of the graphs

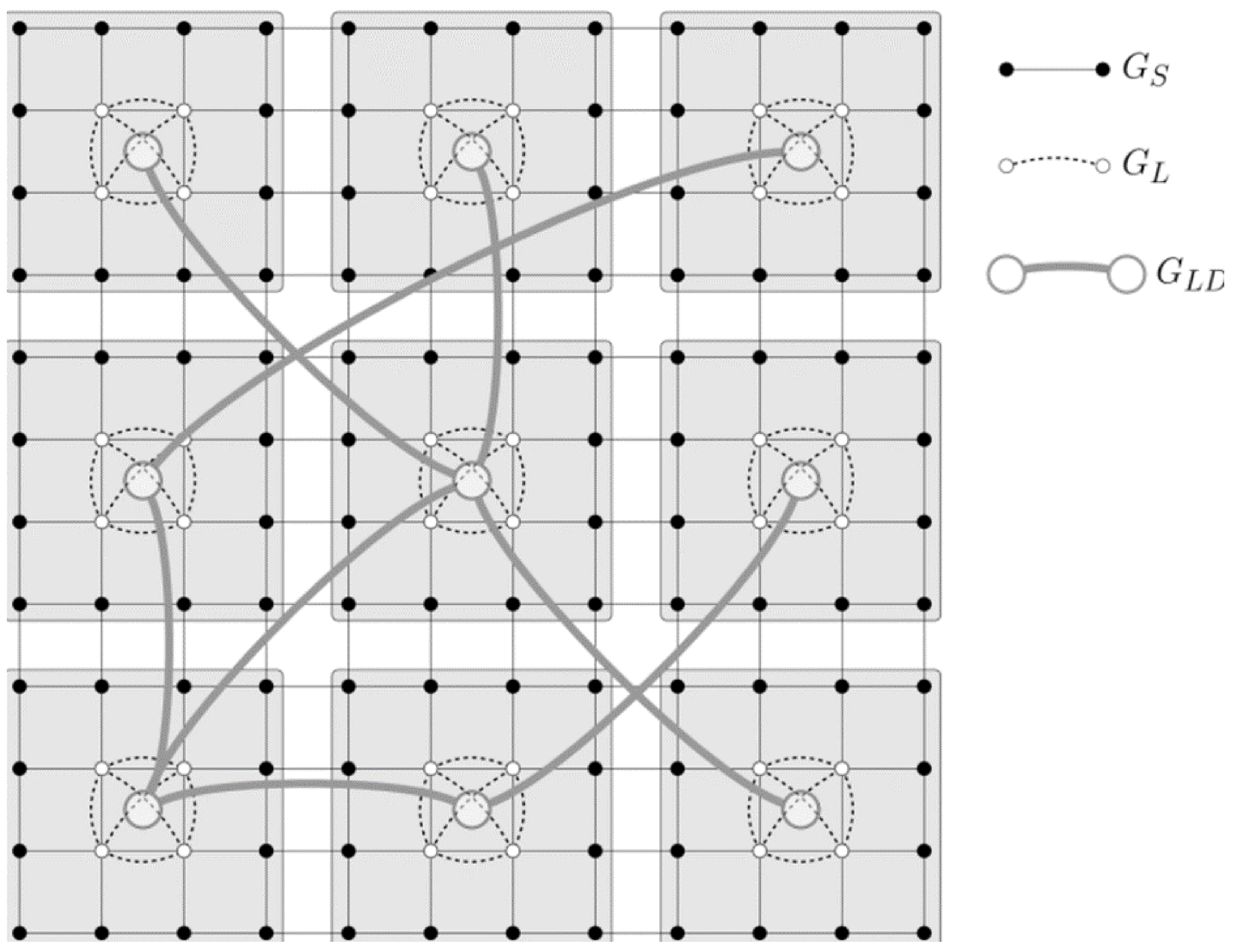

Figure #2

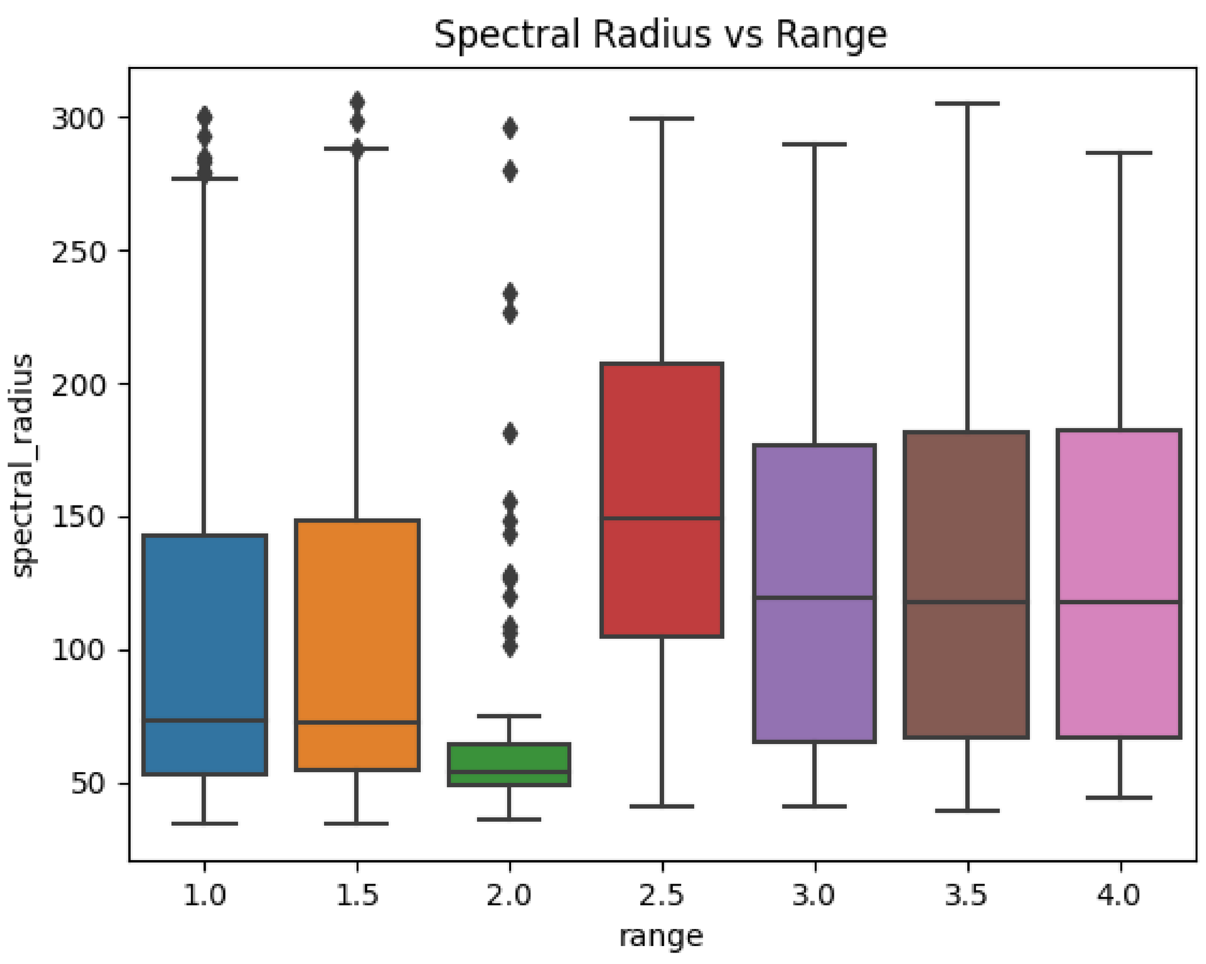

Figure #4

Citations

- Saha, S., Adiga, A., Prakash, B. A., & Vullikanti, A. K. (2015). Approximation algorithms for reducing the spectral radius to CONTROL epidemic spread. Proceedings of the 2015 SIAM International Conference on Data Mining. https://doi.org/10.1137/1.9781611974010.64
- McNitt, J., Chungbaek, Y. Y., Mortveit, H., Marathe, M., Campos, M. R., Desneux, N., Brévault, T., Muniappan, R., & Adiga, A. (2019). Assessing the MULTI-PATHWAY threat from an invasive Agricultural PEST: Tuta absoluta in Asia. Proceedings of the Royal Society B: Biological Sciences, 286(1913), 20191159. https://doi.org/10.1098/rspb.2019.1159